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ABSTRACT

Following a review of studies that propose ways of dealing with the problems that students have in reading mathematics, this paper suggests that the problems need to be approached in basically nontraditional ways. It proposes that in addition to useful, generalized approaches such as readability studies, vocabulary learning, and recognition of complex relationships, educators should explore text and illustrative material relationships, the effects of word frequency and repetition, and the influence of objectivity on the reading task to help students learn to read mathematics. (FL)

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Decoding the Calculus: Double Jeopardy in Readability

Philip M. Rubens

Since we entered the Space Age there has been an increasing emphasis on the sciences and mathematics. Coupled with that emphasis has grown a suspicion that students do poorly in these subjects simply because they can't read well. The only exception to this viewpoint, a remarkable and recent one, is that perhaps the textbooks in specific content areas are beyond the student's reading abilities. Whichever position one takes the central problem still remains--students in science and mathematics courses have difficulty understanding their textbooks.

While it would certainly be interesting to explore this problem in its broadest sense, time and personal inclination will limit this discussion to reading and mathematics. For a very significant reason. Mathematics is the basic tool required for success in all scientific and technological studies.

Numerous researchers have attempted to pinpoint the exact areas which trouble mathematics students and to suggest ways in which such trepidations could be overcome. James and Tanis Bryan, for instance, explore the relationship of these difficulties to learning disabilities.¹ They point out that such disabilities effect spatial orientation, the perception of visual symbols, relationships requiring the ability to associate visual and verbal stimuli, and many other learning patterns. A number of remediation techniques which address these problems will be discussed later.

Other researchers have attempted a variety of empirical studies to isolate some of the particular reading problems that are generally cited as major hindrances to understanding mathematics. Russell Call

and Neal Wiggin, for example, evaluated the effectiveness of teaching reading in the mathematics classroom by assigning one class to a normal regimen (Math teacher, text, and lecture without formal reading instruction) and another to an experimental method (English teacher with limited reading training, text, and lecture with specific instruction in translating words into mathematical symbols). Since the experimental group performed better than the control group, the logical inference is that teaching reading, at least in this content area, has a significant influence on the student's acquisition of usable information.²

All of these researchers keep circling around the same problems, examining them like a diamond cutter looking for the right plane to strike. All of these researchers are also doing, without a doubt, precisely what needs to be done. Questioning, probing, selecting, even guessing about the nature and features of an intriguing reading problem.

If one attempts to winnow the basic features of mathematics-reading problems out of all this research, he or she would have in hand a very small list. Basically it would have three or four problems at best: vocabulary, symbols, complexity of ideas, and level of difficulty. While this looks like a workable number of problems, it is little comfort for the student who has just opened a door and discovered another universe peopled with grotesque creatures he or she must somehow understand.

Vocabulary in both mathematics and science constantly changes. In our technological community word formation goes on by the addition of prefixes and suffixes to root words as well as by combining entire words in unfamiliar patterns. Some, but not all, of this

lexical activity is warranted. In fact, Albert Einstein once said that "Most of the fundamental ideas of science are essentially simple, and may, as a rule, be expressed in a language comprehensible to everyone."³ Textbooks as well as scholarly works tend to belie this observation. Students quickly discover that they are dealing with a highly specialized, almost foreign, vocabulary. The tendency, of course, is for the student to read the text for "cues" and hope that the definition comes to them by a strange process something like osmosis. One would hardly consider using a dictionary, which might not have the definition anyway.

Several researchers have examined the importance of acquiring an adequate vocabulary to read a mathematics text. Mary C. Austin sees vocabulary acquisition as well as the ability to develop concepts as central to understanding mathematics. The student, in her view, must be able to understand the language and to translate or decode it.⁴ Her assertions are born out in the work of John Curney and William Fosinger. These researchers achieve a high degree of success among students who were deliberately and methodically introduced to specialized vocabulary before reading their textbooks.⁵

Like any other field the vocabulary of mathematics is in a constant state of flux. In Calculus, for instance, the direction has completely changed since Newton. The earliest Calculus texts were in Latin and were largely concerned with explaining the actions of the macroscopic world--the movement of planets and the effects of that motion. Today, Calculus texts are written in the vernacular and are interested predominately in describing the microscopic (even sub microscopic) world--the paradoxical and enigmatic properties of light, and the nature of the atom. Newton would hardly recognize

his work, nor would he be able to read or understand it. It is little wonder that students have problems with such works.

Another problem area involves the interpretation of symbols, examples, and illustrative materials as well as evaluating their relationship to the text. One explanation for this problem has been that dyslexic readers have difficulty in perceiving the visual symbols and in associating the visual and the verbal in any given text.⁶ Interestingly, when students are questioned about the relationship between visuals and the text they generally respond that the visuals are too esoteric; they have little to do with the text. Faculty, in contrast, generally see exempla as trite and unimaginative. Obviously, there is a wide variation in expectations and inclination operating in this instance.

A variety of approaches have been suggested for solving this problem. Leo Fay, for example, simply rewrites the SQ3R to SQRQCQ (Survey, Question, Read, Question, Compute, Question) and suggests the latter formula as a palliative for reading mathematics.

A similar proposal by James Riley and Andrew Pachtman divides the reading of mathematics into three levels--literal, interpretive, and applied. They, however, do not see these as mutually exclusive levels; instead, they maintain that success in mathematics is predicated on the reader's ability to "shuttle" between all of these levels.⁸ Perhaps the most telling argument against the last two works is that of Kenneth Henderson who argues that symbolic representations--words, visuals, math notations, numbers--act as "cues" which the capable reader can use to decode the text. Whether a student confronts prose or mathematics, Henderson maintains that their problems are always the same; the content area

has little to do with the nature of the problem.⁹ Such an approach, while interesting, avoids the issue. Readers, especially in mathematics, do have difficulty discerning the relationship between the text and illustrative material.

A third problem area is the level of difficulty in the writing itself. Mathematics by its very nature has a strong symbolic orientation. Once again, it was Einstein who said that the most difficult part of his work in physics was translating symbol into prose.¹⁰ Admittedly, this is a hard task. Witness, for instance, this passage:

"The question of precisely how many boundary conditions, and of what nature, are to be associated with a given partial differential equation to assure the existence and uniqueness of a solution is a question of considerable difficulty. In this book, we shall use the most popular practical guide, physical intuition. Existence is no problem. We seek a solution and if we find one, we know it exists; if we do not find one, it is hardly balm for our wounds to be told that it exists. To the serious problem of uniqueness we close our eyes entirely."

How does a student react to such speculative thinking? How does a student relate this to mathematics? to the application of this material to other courses in science and engineering? There seems to be a rule in the academic community which states that as ideas become more complex, more ambiguous, writing to explain those ideas must also become complex and obscure.

In the sample passage, for instance, the kernel sentence in the first complete sentence is "question...is a question." To understand

this sentence the student must overcome this ambiguity. As many researchers have demonstrated this is no easy task and, in fact, may be insurmountable for many readers.¹² It should be remembered, however, that this sense of ambiguity has been introduced by the author as well as by the author's reaction to his or her material. For many writers complex ideas implies complex writing. Such a stance does little to help readers.

This sense of complexity in ideas is another issue that plagues mathematics readers. While a reader tries to decode prose passages, relate them to illustrative material, and understand new vocabulary, he or she must also deal with complex ideas that often seem at odds with the real world. For instance, when Werner Heisenberg and Neils Bohr were trying desperately to explain some of the early discoveries in quantum mechanics, Heisenberg says that he often found himself speculating "Can nature possibly be as absurd as it seemed to us in these atomic experiments."¹³ If even the experts have trouble understanding their areas as well as difficulty explaining their findings, how can the lay person or novice be expected to comprehend these issues in any form.

Until this point the major thrust of this paper has been to enumerate problem areas and to review relevant arguments concerning reading mathematics. Now it is time to examine some suggestive approaches to this problem.

First, it is necessary to recognize that a great deal of the methodology presently used to help alleviate reading problems in mathematics have their use and validity. Comparative studies of textbook grade levels and student reading ability certainly give teachers valuable information for making judicious decisions in

selecting textbooks.¹⁴ It is also necessary to appreciate the efforts of those who have offered a variety of formuli and guidelines for teaching in this content area.¹⁵ Oftentimes, as in the work of James Riley and Andrew Pachtman, these efforts offer some insights into new and exceptionally useful techniques. In their work, for instance, they describe the "shuttling" techniques necessary for success in mathematics. They also demonstrate that this success occurs only when the student perceives that reading and problem solving are an integrated process rather than a series of discreet and unrelated steps.

Beyond this initial recognition of the validity and worth of these efforts lies a wide range of other, often neglected, interests that effect reading in mathematics. In the sample passage, for instance, the kernel sentence "question...is a question" can be used to examine the effect of word repetition. This effect occurs in relating complex ideas simply because many words for expressing such ideas do not have cognates. Therefore, in mathematics a radical is always a radical and an N-polynomial problem is always an N-polynomial problem. The result, of course, is a sense of boredom, monotony, and repetition for the reader. Another aspect of this effect is that many of these terms occur very infrequently in an average person's vocabulary; consequently, decoding the term itself interferes with the reading process.¹⁶

Another area that needs more investigation is the use of illustrative material. Frederick Hollander makes some exceptionally cogent suggestions concerning the relation of such material to the text as well as the reader's reaction to this relationship. Symbols, exempla, and visuals, he maintains need to be viewed by the writer

as shorthand to succinctly describe a larger issue. This larger issue should be treated extensively in the text first and then expanded upon symbolically later. In his scheme there is a one to one relationship between text and illustrative material. Such a system should answer student objections concerning the irrelevance of exempla. This sense of correspondence between text and illustrative material gives rise to two additional issues. One, without a coherent, logical, and apparent ordering principle to control such a correspondence the student will experience a high degree of frustration and anxiety.¹⁷ A second issue is the need to train writers in methods for judicious selection of illustrative material. This latter issue, of course, has no easy solution; in fact, many textbook authors seem to be looking over someone else's shoulder when choosing examples. These issues--repetition, word frequency, and the uses of illustrative material--represent some of the key problems encountered in reading mathematics. Problems, which for one reason or another, have been neither systematically examined in terms of their effect on readability nor corrected in the writing stages. Since they are essentially mechanical problems they could be treated in the instructional concerns of both writing and reading classes.

A final group of approaches to readability in mathematics (and perhaps in all areas) concerns the reasons readers read and their relationship to the text. While mathematics, science, and technology cling to the tenets of objectivity, considerable evidence has been amassed refuting these same tenets. This is hardly the forum for arguing the nature of subjectivity versus objectivity; however, parts of this dialectic are particularly relevant to reading.¹⁸

In terms of reading concerns this refutation of objectivity begins with Abraham Maslow who maintains that:

The picture of truth and of reality that we have inherited from the classical science of the impersonal is that it is "out there," perfect, complete, hidden but uncoverable. In the earlier versions the observer simply observed. In later versions it was understood that the observer had spectacles that distorted but which could never be removed. Most recently physicists and psychologists have learned that the act of observation is itself a shaper, a changer, an intruder into the phenomenon being observed.¹⁹

Modern physics, then, sees the observer as in some degree an active agent in the observing act, as somehow influencing the outcome of events. The subjective and objective world view seem to have less and less credibility as we explore the sub atomic universe. A step behind Maslow, and in another area, is the work of Maurice Merleau-Ponty who suggests the close relationship of subjective and objective in language: "Words are never fully determined in their abstract signification but have meaning only with relation to man's body and to its interaction with its surroundings."²⁰ What Merleau-Ponty touches on is a general feeling that not only is science somehow "subjective" but all of language as well.²¹

This general drift in contemporary thinking about the nature of objectivity has far-reaching implications for reading, especially in science and mathematics. Of considerable interest for reading theory is the work of Louise Rosenblatt. Her basic tenet seems particularly relevant to reading objective material. She

maintains that each reader participates in a process of "selective attention" which leads the reader to concentrate on a specific mode of reading--efferent or aesthetic. In the former mode--efferent--the reader reads to gather information; in the latter, for enjoyment of a work of art. This neat division does not always work for objective information; some people do read such material from the aesthetic stance. However, this concept does neatly categorize ways in which one may look at the reading task.

What then can one garner from this discussion? In general, we need to realize that the reading of mathematics (and indeed much objective prose) has to be approached in some basically non-traditional ways. In addition to useful generalized approaches--readability studies, vocabulary learning, recognition of complex ideas--it is essential that we explore text and illustrative material relationships, the effects of word-frequency and repetition, and the influence of objectivity on the reading task. Admittedly, these are all somewhat grayer areas, all far more esoteric than we normally explore. However, the groundwork has been established; all we need do is utilize it.

NOTES

- ¹James & Tanis Bryan. Understanding Learning Disabilities. pp. 214-235. This work was of considerable importance to this study because of the unusually high percentage of dyslexia cases in our student body.
- ²Russell Call and Neal Wiggin. "Reading and Mathematics," Mathematics Teacher, (Feb. 1966), pp. 149-57.
- ³Albert Einstein and Leopold Infeld. The Evolution of Physics, NY: Simon & Schuster, 1938, p. 27.
- ⁴Mary C. Austin. "Improving Comprehension of Mathematics," Reading in the Secondary Schools, ed. M. Jerry Weiss, NY: Odyssey Press, 1961, pp. 391-396.
- ⁵John J. Carney and Wm. Fosinger. "Reading and Content in Technical-Vocational Education," Journal of Reading, Vol. 20 (Oct. 1976), 14-17.
- ⁶Bryan, pp. 219, 225-226.
- ⁷Leo Fay. "Reading Study Skills: Math and Science," International Reading Association: Proceedings of the Annual Convention, Vol. 10 (1965), 93.
- ⁸James D. Riley and Andrew B. Pachtman. "Reading Mathematical Word Problems: Telling Them What To Do Is Not Telling Them How To Do It," Journal of Reading, Vol. 21 (March, 1978), 532-533.

- ⁹Kenneth B. Henderson. "Interpreting Materials in Arithmetic and Mathematics in Grades Seven To Fourteen," in M. Jerry Weiss, ed., Reading in the Secondary Schools. NY, Odyssey Press, 1961, pp. 118-120.
- ¹⁰Readers should not interpret my references to Einstein as an isolated example of the technologists concern for language. One could as easily point to such figures as Werner Heisenberg ("Even for the physicist the description in plain language will be a criterion of the degree of understanding that has been reached.") or Erwin Schrödinger ("If you cannot--in the long run--tell everyone what you have been doing, your doing has been worthless.")
- ¹¹Rainville and Bedient. Elementary Differential Equations. NY: Holt, Rinehart, Winston, 1975, p. 406.
- ¹²D. G. MacKay. "To End Ambiguous Sentences," Perception and Psychophysics, Vol. 1 (1966), 426-436; cf., Donald G. MacKay and Thomas G. Bever, "In Search of Ambiguity," Perception and Psychophysics, Vol. 2 (1967), 193-200; cf., Donald J. Foss, "Some Effects of Ambiguity upon Sentence Comprehension," Journal of Verbal Learning and Verbal Behavior, 9 (1970), 699-706.
- ¹³Werner Heisenberg. Physics and Philosophy. NY: Harper & Row, 1958, p. 42.
- ¹⁴Maurice Kurzman. "The Reading Ability of College Freshmen Compared to the Readability of Their Textbooks," Reading Improvement, Vol. II (Fall 1974), 13-25; cf., Dorinda and Leslie McClellan,

"A Comparison of the Readability Level of Text Materials with the Reading Level of Community College Students," ERIC; cf., Elaine E. Cherney, Relationship of Reading Ability of Remedial Track University Freshmen to Text Readability and Instructional Methodology. Unpublished PhD Diss, Michigan State University, 1975.

¹⁵Fay, Riley, and Carney are good examples of such efforts. Another more general approach is offered by Richard T. Vaca, "The Development of a Functional Reading Strategy: Implications for Content Area Instruction," The Journal of Educational Research, Vol. 69 (Nov. 1975), 108-112.

¹⁶D. E. Broadbent. "Word-Frequency Effect and Response Bias," Psychological Review, Vol. 74, #1 (January 1967), 1-15; cf., Jack Catlin, "Theoretical Notes on the Word-Frequency Effect," Psychological Review, Vol. 76, #5 (1969), 504-506.

¹⁷Frederick H. Hollander. "The Process of Reading in Mathematics and Science," Western College Reading Association, Vol. 5 (1970), p. 67, 68.

¹⁸One could point to a variety of writers to discuss this argument. For purposes of this paper the works of Herbert Simon, J. Bronowski, David Bleich, and Louise Rosenblatt and particularly relevant.

¹⁹Abraham Maslow. The Psychology of Science: A Reconnaissance. NY: Harper & Row, 1966, pp. 110-111. The more recent manifestations of the act of observation are evident in many facets of quantum mechanics, especially the work Niels Bohr,

Louis De Broglie, Wolfgang Pauli, Max Born, and Werner Heisenberg.

- ²⁰Maurice Merleau-Ponty. Phenomenology of Perception. Trans. Colin Smith. London: Routledge, 1962, pp. 181-184.
- ²¹The empirical work of Patrick M. Rabbitt ("Learning to Ignore Irrelevant Information," The American Journal of Psychology, Vol. 80, #1 (March, 1967), 1-13 and "Ignoring Irrelevant Information," British Journal of Psychology, Vol. 55, (1964), 403-414.) and Donald P. Cleland ("A Construct of Comprehension in Reading and Inquiry," Conference Proceedings, 10 (Newark; Delaware: Intl. Reading Assoc., 1965), 59-64.) support Merleau-Ponty's observations.